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"Propagation of E. m. wave in an isotropic plasma (Ionospheric Reflection)"

Since Ionosphere is also a plasma as it consists ionised gases. when presure is low then there ~~are~~ no atraction between electrons hence no energy losses. so the term Conductivity for inosphere, σ is completly imagenary. and it is given by

$$\sigma = \frac{i n_0 e^2}{m\omega} \quad \text{—— (1)}$$

where n_0 = no. of electrons for unit volume

For an ionise medium

$$\rho = 0, \quad \mu = \mu_0 \;\&\; \epsilon = \epsilon_0$$

Maxwell's four fields eqns are

$$\vec{\nabla}\cdot\vec{D} = 0 \quad \text{or} \quad \vec{\nabla}\cdot\vec{E} = 0 \quad (\because \vec{D} = \epsilon_0 E) \quad \text{—— 2(a)}$$

$$\vec{\nabla}\cdot\vec{B} = 0 \quad \text{or} \quad \vec{\nabla}\vec{H} = 0 \quad (\because \vec{B} = \mu_0\vec{H}) \quad \text{—— 2(b)}$$

$$\text{Curl}\,\vec{E} = -\frac{\partial\vec{B}}{\partial t} = -\mu_0\frac{\partial H}{\partial t} \quad \text{—— 2(c)}$$

$$\text{Curl}\,\vec{H} = j + \frac{\partial\vec{D}}{\partial t} = \sigma\vec{E} + \epsilon_0\frac{\partial\vec{E}}{\partial t} \quad \text{—— 2(d)}$$

—— (2)

Now, taking curl of 2(c)

$$\text{curl curl } \vec{E} = \mu_0 \frac{\partial}{\partial t}(\vec{\nabla} \times \vec{H}) \quad \text{--- (3)}$$

$$\vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \nabla^2 E = -\mu_0 \frac{\partial}{\partial t}(\vec{\nabla} \times \vec{H}) \quad \text{--- (4)}$$

using 2(a) and 2(d) in eqn (4) we get

$$0 - \nabla^2 E = -\mu_0 \frac{\partial}{\partial t}\left(\sigma \vec{E} + \epsilon_0 \frac{\partial \vec{E}}{\partial t}\right)$$

$$\nabla^2 \vec{E} = -\mu_0 \sigma \frac{\partial \vec{E}}{\partial t} - \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t^2} \quad \text{--- (5)}$$

$$\nabla^2 \vec{H} - \mu_0 \sigma \frac{\partial \vec{H}}{\partial t} = \mu_0 \epsilon_0 \frac{\partial \vec{H}}{\partial t^2} \quad \text{--- (6)}$$

Both eqn (5) & eqn (6) will satisfy the same scalar wave eqn

$$\nabla^2 \psi - \mu_0 \sigma \frac{\partial \psi}{\partial t} - \mu_0 \epsilon_0 \frac{\partial^2 \psi}{\partial t^2} = 0 \quad \text{--- (7)}$$

where ψ = the scalar function

The plane wave soln of eqn (5) (6) and (7) can be given by

$$\vec{E} = \vec{E}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)} \quad \text{--- (8)}$$

$$\vec{H} = \vec{H}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)} \quad \text{--- (9)}$$

$$\psi = \psi_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)} \quad \text{--- (10)}$$

$$\vec{K} = K\hat{n} \quad \text{(propagation vector)}$$
$$= \frac{2\pi}{\lambda}\hat{n}$$
$$= \frac{2\pi n}{\lambda n}\hat{n} = \frac{\omega}{v}\hat{n} \quad \text{——— (11)}$$

$\vec{v}$ = phase velocity

Substituting equ^n (10) in equ^n (7)

$$\nabla^2\psi = -K^2\psi, \quad \frac{\partial\psi}{\partial t} = -i\omega\psi$$

and $$\frac{\partial^2\psi}{\partial t^2} = -\omega^2\psi$$

we get from equ^n (7)

$$-K^2\psi + \mu_0 i\omega\sigma\psi + \mu_0\epsilon_0\omega^2\psi$$
$$-(K^2 - i\omega\mu_0\sigma - \mu_0\epsilon_0\omega^2)\psi = 0$$
$$\therefore \quad K^2 - i\omega\mu_0\sigma - \mu_0\epsilon_0\omega^2 = 0 \quad \text{——— (12)}$$

or, $$K^2 = \mu_0\epsilon_0\omega^2 + i\omega\mu_0\sigma$$
$$= \mu_0\epsilon_0\omega^2\left(1 + \frac{i\cancel{\omega}\cancel{\mu_0}\sigma}{\cancel{\mu_0}\epsilon_0\omega^{\cancel{2}}}\right)$$
$$= \mu_0\epsilon_0\omega^2\left(1 + \frac{i\sigma}{\epsilon_0\omega}\right) \quad \text{——— (13)}$$

or, $$K^2 = \mu_0\epsilon_0\omega^2\left(1 + \frac{i}{\epsilon_0\omega} \times \frac{in_0e^2}{m\omega}\right)$$
(using equ^n (1))

or, $$K^2 = \mu_0\epsilon_0\omega^2\left(1 - \frac{n_0e^2}{m\epsilon_0\omega^2}\right) \quad \text{——— (14)}$$

or, $K^2 = \mu_0 \epsilon_0 \omega^2 \left(1 - \frac{\omega_p^2}{\omega^2}\right)$ —— (15)

where $\frac{n_0 e^2}{m \epsilon_0} = \omega_p$ = plasma angular frequency. —— (16)

$\because K = \frac{\omega}{v} = \frac{\omega}{c/n} = n \frac{\omega}{c}$

$\therefore K = \frac{n \omega}{c}$ —— (17)

$K^2 = \frac{n^2 \omega^2}{c^2}$ —— (18)

equating equn (15) & (18) we have

$\frac{n^2 \omega^2}{c^2} = \mu_0 \epsilon_0 \omega^2 \left(1 - \frac{\omega_p^2}{\omega^2}\right)$

$n^2 = \left[1 - \frac{\omega_p^2}{\omega^2}\right]$ —— (19)

$n = \left[1 - \frac{\omega_p^2}{\omega^2}\right]^{1/2}$ —— (20)

n = R.I. of the ionosphere

(i) If $\omega > \omega_p$ then R.I. n will be real value.

Thus e.m. waves propagate freely through inosphere.

(ii) If $\omega < \omega_p$ R.I n will be imaginary and e.m. wave ~~~~ incident on plasma will be reflected from the surface.

$$\boxed{k = \frac{in\omega}{c}} \quad \text{——— (21)}$$

Skin depth for plasma

$$\delta_{plasma} = \frac{1}{\beta} = \frac{1}{n\omega/c}$$

$$\delta_{plasma} = \frac{c}{n\omega}$$

$$\delta_{plasma} = \frac{c}{\sqrt{\omega_p^2 - \omega^2}} \quad \text{——— (22)}$$

$\omega << \omega_p$

$$\delta_{plasma} = \frac{c}{\omega_p} \quad \text{——— (23)}$$

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